Subject: *Differential Geometry*

Subject Code: MDSE(M) 404A

M.Sc. Mathematics Fourth Semester

Curves in Space Curvature

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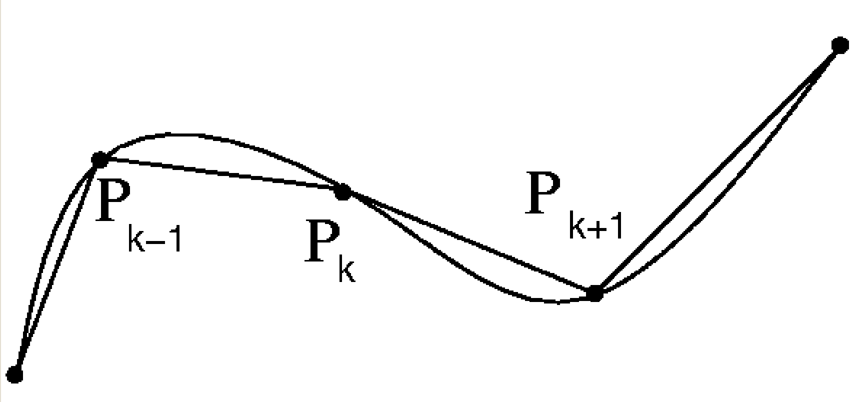
Description of Plane Curves

- Explicit form: $y = f(x)$
- Implicit form: $F(x, y) = 0$
- Parametric Form: $\vec{r}(t) = [x(t), y(t)]$

Arc Length



Arc length is curvilinear distance along curve

$$\sum_k \sqrt{(x_{k+1}-x_k)^2+(y_{k+1}-y_k)^2} \rightarrow \int \sqrt{(dx/dt)^2+(dy/dt)^2}$$

Arc length $s(t)$

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$$

$$s(t) = \int_{t_0}^{t} \sqrt{(dx/dt)^2+(dy/dt)^2}$$

Tangent and Normal Vectors.

$$\vec{t}(s) = \frac{d\vec{r}(s)}{ds}$$

Unit speed parameterization

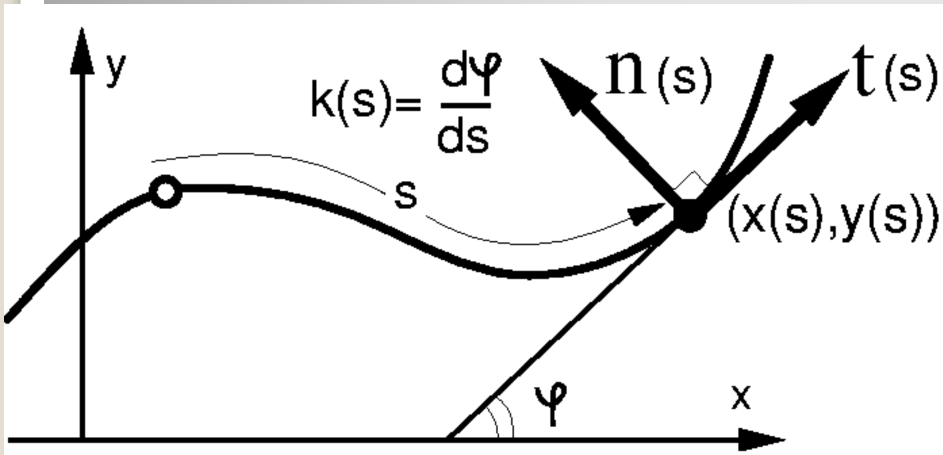
$$\left\|\vec{t}(s)\right\| = 1$$

$$\vec{n}(s) = \frac{d\vec{t}(s)}{ds} \perp \vec{t}(s)$$

Local frame

$$\left[\, \vec{t}(s), \vec{n}(s) \,\right]$$

Arc Length PRepersentation



$$\vec{r}(t) = [x(t), y(t)]$$

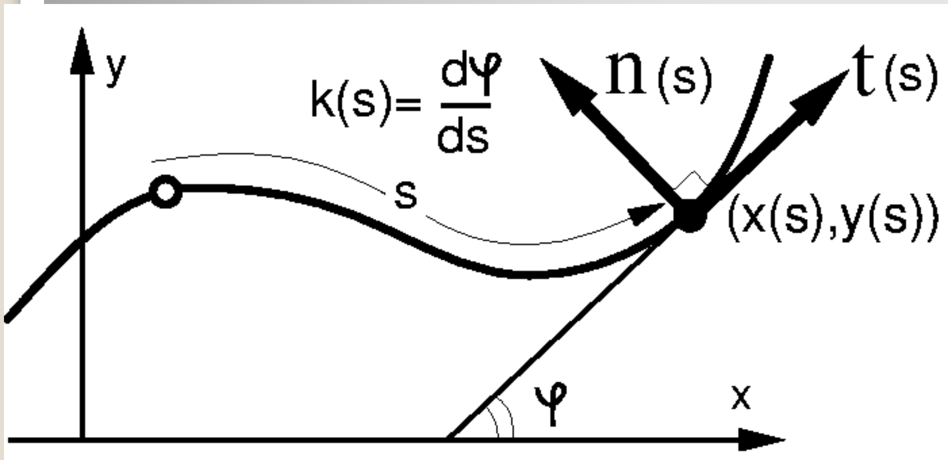
$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$\frac{d\vec{r}}{ds} = \frac{d\vec{r}}{dt}\frac{dt}{ds} = \frac{\frac{d\vec{r}}{dt}}{\left\|\frac{d\vec{r}}{dt}\right\|} \Rightarrow \left\|\frac{d\vec{r}}{ds}\right\| = 1$$

Unit speed parameterization

$$\frac{d\vec{r}}{ds} = \vec{t}(s), \quad \left\|\vec{t}(s)\right\| = 1$$

Curvature, Curvature Vector



$$k(s) = \frac{d\phi(s)}{ds}$$

$$\vec{k} = k(s)\vec{n}(s)$$

The curvature at a point measures the rate of curving (bending) as the point moves along the curve with unit speed

When orientation is changed the curvature changes its sign, the curvature vector remains the same

- Straight line: $k \equiv 0$
- circle oriented by its inner normal : $k \equiv 1/R$

Curvature Profile

curvature vectors

$$\vec{k} = k(s)\vec{n}(s)$$

(curvature profile)

It helps to evaluation curve quality of vectors in space.

Taylor Series Expansion

$$f(x)=\begin{cases} f(x_0)+(x-x_0)f'(x_0)+\dfrac{1}{2}(x-x_0)^2 f''(x_0)+ \\ +\dfrac{1}{6}(x-x_0)^3 f'''(x_0)+\ldots+\dfrac{1}{n!}(x-x_0)^n f^{(n)}(x_0)+\ldots \end{cases}$$

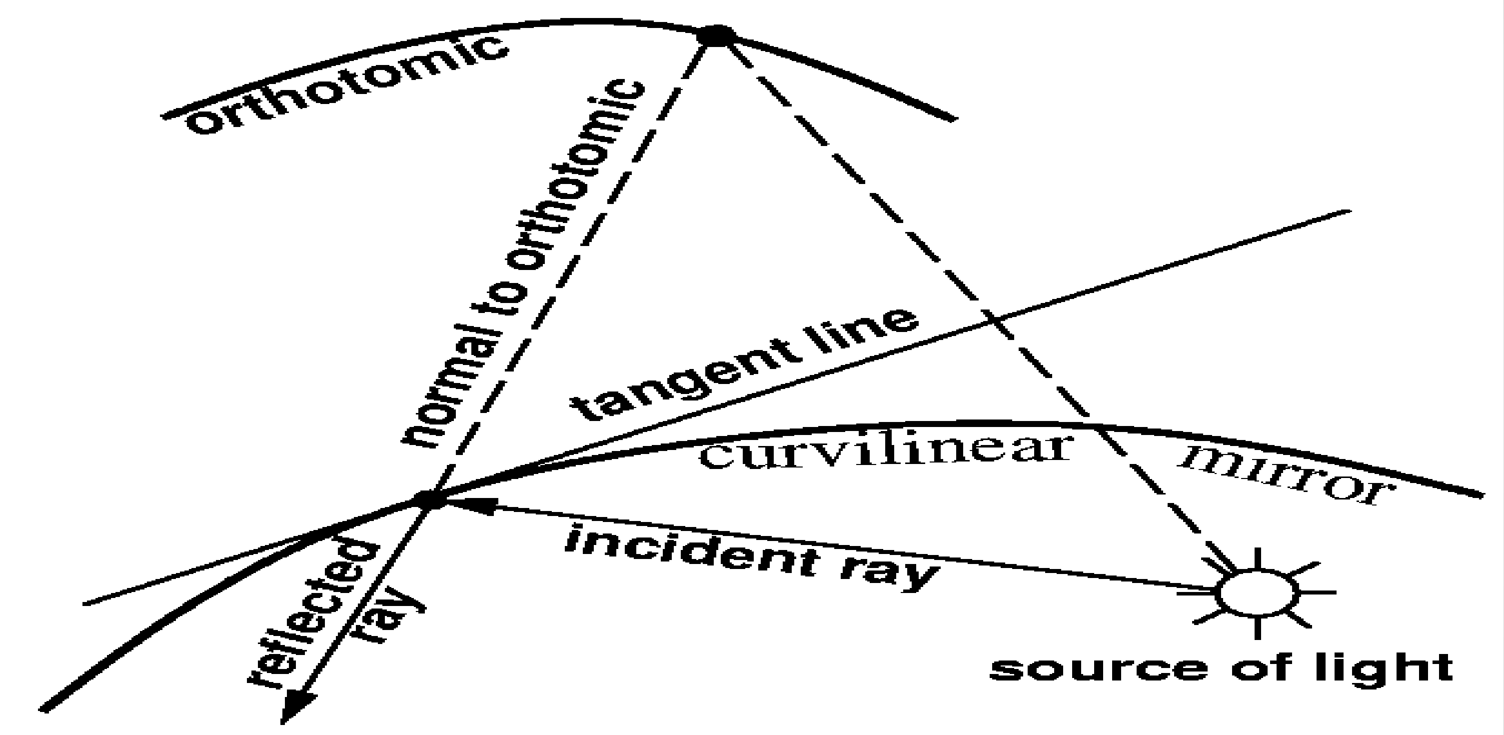
$$\vec{r}(s+\alpha)=\vec{r}(s)+\alpha\vec{r}'(s)+\frac{\alpha^2}{2}\vec{r}''(s)+\ldots$$

$$\vec{r}'=\vec{t},\ \ \vec{r}''=\vec{t}'=k\vec{n},$$

$$\vec{r}'''=\vec{t}''=(k\vec{n})'=k'\vec{n}-k^2\vec{t},\ \ldots$$

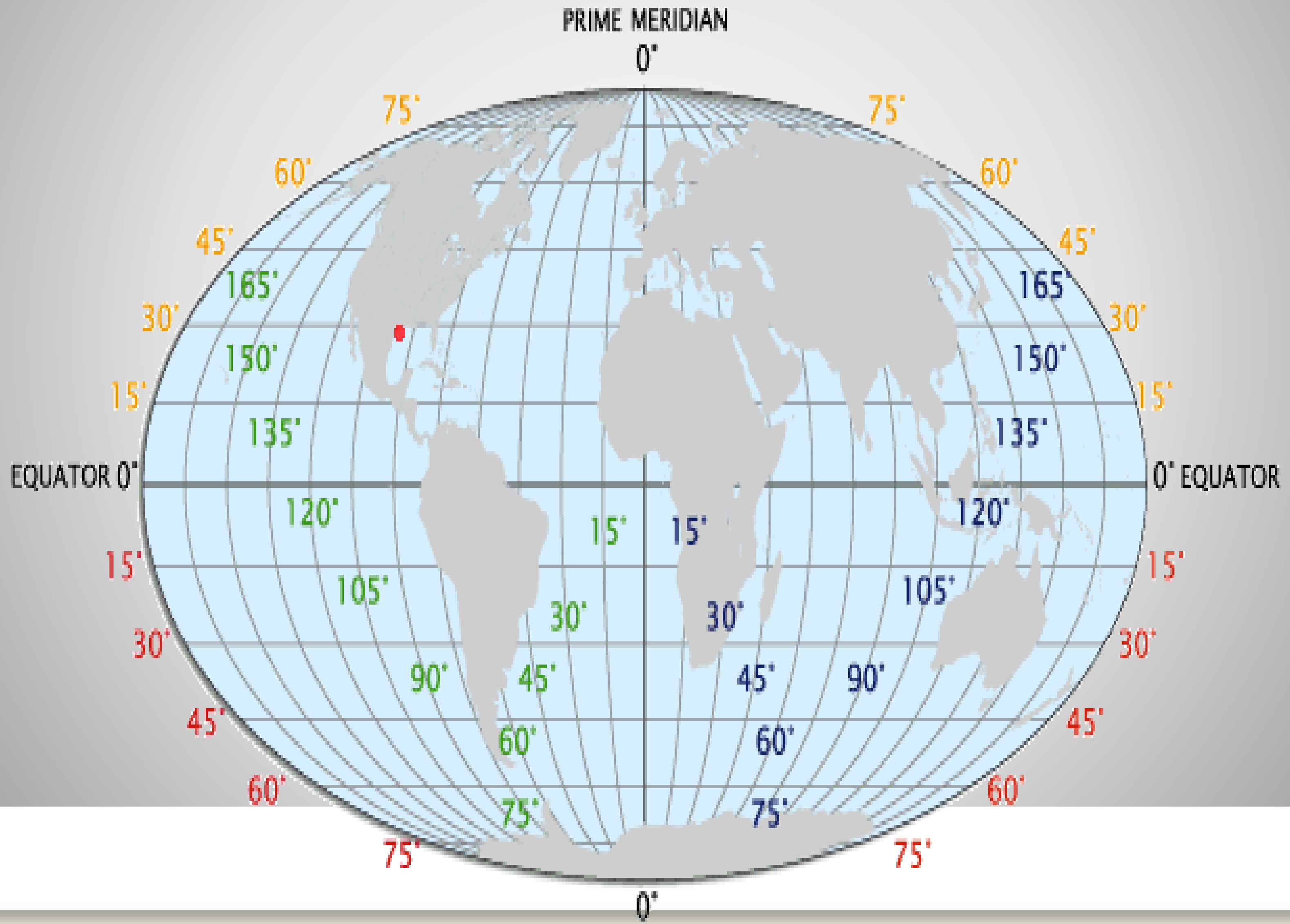
$$\vec{r}(s+\alpha)=\vec{r}(s)+\vec{t}\left(\alpha+\ldots\right)+\vec{n}\left(k\alpha^2/2+\ldots\right)$$

Definition of orthotomic



Given a curvilinear mirror and a source of light, the caustic generated by the light rays reflected by the mirror is the evolute of the orthotomic

How to Determine Longitude



Latitude is the angular distance, in degrees, minutes, and seconds of a point north or south of the Equator. Lines of latitude are often referred to as parallels.

Longitude is the angular distance, in degrees, minutes, and seconds, of a point east or west of the Prime (Greenwich) Meridian. Lines of longitude are often referred to as meridians.

Longitude

Latitude can be calculated from the position of Polaris

Longitude:

- carry a clock along on board ship
- set to Greenwich time (for example)
- at sea, note the time that the sun was at its zenith (i.e. local noon) using this clock
- compare to the compiled information.

Thanking You